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The Identity of Numbers: From Arithmetic Invariants to Mathematical Structuralism

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Abstract: What makes a number the same number across different representations and mathematical realizations? This paper examines numerical identity at the intersection of number theory and philosophy of mathematics. It distinguishes numeral representation, arithmetic structure, and mathematical ontology. The integer 30, for example, may be represented as 30, XXX, or 11110_2 , while its arithmetic properties remain unchanged. Moving beyond notation, the paper considers isomorphic realizations of the natural numbers and shows explicitly that arithmetic properties such as primality, divisibility, and unique factorization are preserved under arithmetic isomorphism. This provides a precise mathematical basis for asking what, philosophically, constitutes numerical identity. Benacerraf's identification problem then shows why arithmetic does not privilege a particular set-theoretic realization of numbers. The paper argues that these mathematical facts strongly support methodological structuralism - the practice of identifying mathematical objects through structure-preserving relations - without thereby establishing metaphysical structuralism as the unique ontology of mathematics. Computer-assisted proof and artificial intelligence are considered as a contemporary extension of the same distinction between formal preservation and mathematical understanding. The resulting thesis is deliberately interdisciplinary: arithmetic invariance constrains philosophical accounts of number while leaving the ultimate ontology of mathematical objects open.

Keywords: Numerical identity, Arithmetic invariants, Number theory, Mathematical structuralism

Introduction

Numbers are mathematically elementary and philosophically difficult. Arithmetic can be practiced successfully without a settled answer to the question 'What is a number?', yet any philosophical account of arithmetic must explain why different symbols, constructions, and implementations can be treated as concerning the same numerical objects. This paper treats that tension as a problem of numerical identity. Its primary concern is not the historical development of numeral systems but the mathematical features that remain stable when representations and realizations change.

Three stones, three sounds, the numeral 3, and a set-theoretic representative of 3 are not the same kind of thing. The first two are collections or events with cardinality three; the third is a sign; the fourth is a possible foundational realization. Conflating these levels obscures the philosophical problem. The relevant question is what warrants treating such different representations or realizations as concerning the same number. Benacerraf's identification problem makes this difficulty especially clear. Different set-theoretic constructions can realize the natural numbers while disagreeing about the internal membership properties of the objects assigned to particular numerical positions. Arithmetic nevertheless proceeds unchanged. Structuralist approaches take this as evidence that mathematical practice privileges positions, relations, and structure-preserving transformations over the accidental constitution of a chosen implementation (Benacerraf, 1965; Shapiro, 2000; Linnebo, 2017).

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The central thesis is deliberately two-sided. First, arithmetic exhibits robust invariance across legitimate changes of representation and across isomorphic realizations. Second, this mathematical invariance does not by itself determine a unique ontology of numbers. The paper therefore distinguishes methodological structuralism - a claim about how mathematics identifies and compares objects - from metaphysical structuralism - a stronger claim about what mathematical objects ultimately are.

Research Question and Thesis

The guiding question is: which features of a number are invariant across legitimate representations and realizations, and what philosophical conclusions follow from that invariance? The paper defends three claims. First, representational invariance: changing a numeral or coding need not change the number represented. Second, structural characterization: arithmetic properties are tracked through relations and operations preserved by appropriate isomorphisms and embeddings. Third, ontological underdetermination: these mathematical facts constrain philosophical accounts of number but do not decide whether numbers are abstract objects, positions in structures, logical constructions, or entities understood through another foundational framework.

Method and Scope

This is a conceptual study constrained by mathematical examples. Elementary number theory is used because divisibility, primality, and unique factorization make invariance transparent. Foundational examples are then used to test how far such invariance supports structuralist interpretation. Historical episodes function as conceptual background rather than as an exhaustive history. The discussion of computer-assisted proof and artificial intelligence is similarly limited: it concerns verification, explanation, and mathematical understanding, not a general theory of AI.

Original Contribution

The contribution is a bridge argument rather than a new theorem in number theory. The paper moves from ordinary representational invariance to an explicit arithmetic isomorphism, proves that primality is preserved under that isomorphism, and then uses the result to separate a mathematically well-supported structural methodology from stronger metaphysical conclusions. This formulation connects ordinary number-theoretic practice with philosophical work on invariance and structuralism while avoiding the claim that invariance alone settles mathematical ontology (Carter, 2008; Schiemer, 2014). Figure 1 summarizes the argumentative division of labor: mathematics identifies what is preserved; philosophy asks what, if anything, that preservation implies about the identity and ontology of numbers.



Mathematics establishes what is preserved; philosophy asks what the preservation means.

Figure 1. Argument map: from representation to arithmetic invariance and philosophical interpretation.

Historical Background: From Counting to Abstract Representation

Early counting practices used fingers, pebbles, tally marks, and other material devices to stabilize quantity. Their philosophical significance is limited but important: a counting device is not the number it represents. One-to-one correspondence allows materially different collections to be treated as equinumerous, creating conceptual space for cardinality to be separated from the qualitative nature of counted objects (Boyer & Merzbach, 2011; Joseph, 2011).

Representation and Abstraction

Different cultures developed different numeral systems, bases, and notational practices. This diversity supports a basic distinction between number and numeral: the sign can change while arithmetic relations remain stable. The point is not yet structuralism; it is the more modest observation that numerical identity is not exhausted by inscription.

Greek Proof and Mathematical Necessity

The systematic role of demonstration in the Euclidean tradition added a different dimension. Mathematical claims became answerable not merely to repeated observation but to proof from accepted premises. Philosophically, this shifts attention from recognizing quantitative patterns to explaining why mathematical propositions must hold (Boyer & Merzbach, 2011).

Positional Notation and Zero

The Indian place-value tradition and the use of zero greatly increased the efficiency and generality of numerical representation. Zero as a positional marker should be distinguished from zero as a number governed by arithmetic rules; neither role warrants identifying zero with metaphysical nothingness. The episode instead illustrates how notation can transform mathematical practice without being identical with the objects represented (Joseph, 2011). Mathematics in the Arabic-Islamic tradition further illustrates the movement from particular representations toward general procedures. Al-Khwarizmi's works helped transmit and systematize Hindu reckoning while also treating algebraic problem classes in a general way. For the present argument, the important point is not a simple linear narrative of transmission but the increasing capacity to reason about quantities through general operations and procedures (Rashed, 1994).

From Representation to Arithmetic Structure

The decisive step for the present argument occurs within mathematics itself. A number theorist does not identify an integer by the typography of its numeral. The integer 30 may be written as 30, XXX, or 11110_2 . What survives these changes are arithmetic relations: its position in the order, its divisors, its behavior under addition and multiplication, and its factorization. Representation is therefore dispensable in a way that arithmetic structure is not.

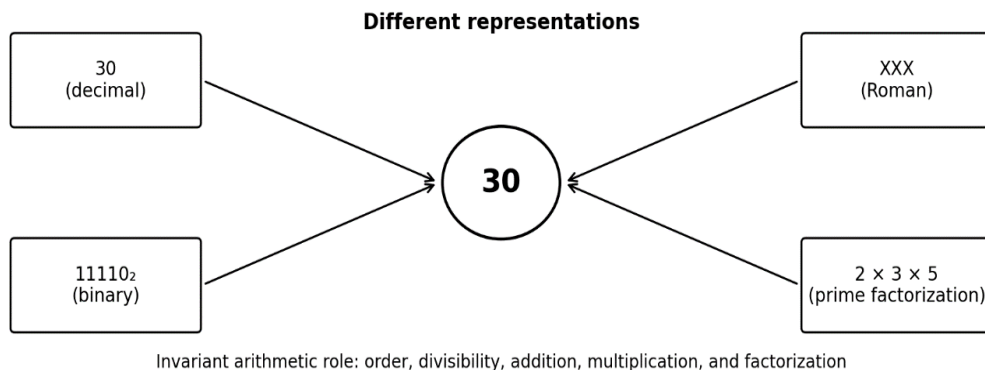


Figure 2. One integer, several representations: what changes and what remains invariant.

Prime Factorization as an Invariant

The Fundamental Theorem of Arithmetic states that every integer greater than 1 has a factorization into primes that is unique up to the order of the factors. In native mathematical notation,

$$n = p_1^{a_1} \cdot p_2^{a_2} \cdot \dots \cdot p_k^{a_k}.$$

For 30, the decomposition is $2 \cdot 3 \cdot 5$. This factorization is independent of whether 30 is written in decimal, binary, Roman notation, or another adequate numeral system. Prime factorization is therefore an arithmetic invariant rather than a property of notation (Hardy & Wright, 2008; Stillwell, 2003).

What the Theorem Does Not Prove

Unique factorization does not establish that numbers are literally structures, positions, sets, or abstract objects. It is a theorem internal to arithmetic. Its philosophical relevance is methodological: mathematical explanation privileges relations that remain stable under legitimate changes of representation. Moving from this fact to an ontology requires an additional argument.

Embeddings and the Extension of Number Systems

A second example comes from the familiar progression of number systems:

$$\mathbb{N} \hookrightarrow \mathbb{Z} \hookrightarrow \mathbb{Q} \hookrightarrow \mathbb{R} \hookrightarrow \mathbb{C}.$$

Each extension addresses a limitation of an earlier domain: integers accommodate additive inverses; rationals accommodate quotients by nonzero integers; reals provide completeness for analysis; and complex numbers provide an algebraically closed field. In ordinary mathematical language these systems are often written as nested inclusions. Foundationally, however, the safer statement is that standard constructions admit canonical structure-preserving embeddings.

Identity Through Preserved Relations

Under these embeddings, the natural number 5 retains the arithmetic relations associated with 5. The surrounding domain and the underlying set-theoretic implementation may change, while the relevant arithmetic role is preserved. This is precisely the kind of practice that motivates structural description.

A Qualification About Isomorphism

Mathematicians frequently work 'up to isomorphism', but the slogan requires qualification. An isomorphism preserves the relations and operations expressed in the relevant structural language; it need not preserve arbitrary properties of the underlying sets used to implement a model. This distinction is essential rather than technical, because it is precisely what gives Benacerraf's challenge its force.

A Worked Structural Example: Two Copies of Arithmetic

Consider the standard natural-number structure

$$\mathcal{N} = (\mathbb{N}, 0, S, +, \times).$$

Here, $\mathbb{N}$ is the set of natural numbers, 0 is the distinguished zero element, and S , $+$, and $\times$ denote successor, addition, and multiplication, respectively. Now construct a second set whose elements are deliberately distinct from the ordinary natural numbers:

$$\mathbb{N}^* = \{0^*, 1^*, 2^*, \dots\} = \{n^* : n \in \mathbb{N}\}.$$

Define arithmetic on this set so that each new element plays exactly the same arithmetic role as its counterpart in

$\mathbb{N}$:

$$S^*(n^*) = (n + 1)^*,$$

$$m^* \oplus n^* = (m + n)^*,$$

$$m^* \otimes n^* = (mn)^*.$$

The second arithmetic structure is therefore

$$\mathcal{N}^* = (\mathbb{N}^*, 0^*, S^*, \oplus, \otimes).$$

Define the correspondence

$$f : \mathbb{N} \rightarrow \mathbb{N}^*, \quad f(n) = n^*.$$

The map f is a bijection and preserves zero, successor, addition, and multiplication:

$$f(0) = 0^*, \quad f(S(n)) = S^*(f(n)),$$

$$f(m + n) = f(m) \oplus f(n), \quad f(mn) = f(m) \otimes f(n).$$

Hence f is an isomorphism of arithmetic structures:

$$\mathcal{N} \cong \mathcal{N}^*.$$

The important point is that n and its corresponding element in the second structure are not literally the same underlying object. Rather, they occupy corresponding positions and satisfy exactly corresponding arithmetic relations. The realizations differ; the arithmetic structure is preserved.

Table 1. Isomorphic realizations of arithmetic and the preservation of arithmetic properties

Standard Arithmetic	Isomorphism f	Disjoint Copy
0	$\mapsto$	0^*
$m + n$ and mn	preserved as	$m^* \oplus n^*$ and $m^* \otimes n^*$
p is prime	$\mapsto$	$f(p)$ prime

Proposition: Primality is Preserved Under Arithmetic Isomorphism

Let f be the isomorphism defined above. For every $p > 1$, p is prime in the standard natural-number structure if and only if $f(p)$ is prime in the disjoint copy.

Proof

Assume first that p is prime. Suppose that $f(p)$ factors in the second structure as

$$f(p) = a^* \otimes b^*.$$

Since f is surjective, there exist $a, b \in \mathbb{N}$ such that $a^* = f(a)$ and $b^* = f(b)$. Because f preserves multiplication,

$$f(p) = f(a) \otimes f(b) = f(ab).$$

Injectivity gives $p = ab$. Since p is prime, either $a = 1$ or $b = 1$. Thus one of the two factors is $f(1)$, the multiplicative identity in the second structure, and $f(p)$ is prime. Conversely, if $f(p)$ is prime, the same argument applied to the inverse isomorphism f^{-1} shows that p is prime. Therefore primality is preserved in both directions. ■

Why This Proposition Matters

The proposition is elementary but philosophically revealing. Primality is not tied to the particular objects chosen to realize the natural numbers. It depends on the multiplication structure and is therefore transported by an arithmetic isomorphism. The same applies to divisibility and to other arithmetic relations definable from the preserved operations.

This gives precise mathematical content to the claim that arithmetic is insensitive to the particular material or coding used to realize its positions (Schiemer, 2014). The conclusion is structural, not unrestricted. An isomorphism need not preserve every property of the underlying objects. Set-theoretic membership properties, for example, may differ between realizations. What is preserved are the relations and operations belonging to the arithmetic structure under consideration.

Corollary: Unique Factorization is Preserved under Isomorphism

Suppose $n > 1$ has the prime factorization

$$n = p_1^{a_1} \cdot p_2^{a_2} \cdot \dots \cdot p_k^{a_k}.$$

Because f preserves multiplication and sends primes to primes, applying f to this equality gives the corresponding factorization of $f(n)$ in the second structure. If powers in the second structure are defined by transport through f ,

$$(f(p))^{\otimes a} := f(p^a),$$

then

$$f(n) = f(p_1)^{\otimes a_1} \otimes f(p_2)^{\otimes a_2} \otimes \dots \otimes f(p_k)^{\otimes a_k}.$$

Thus the Fundamental Theorem of Arithmetic is preserved across isomorphic arithmetic realizations. The invariant is the factorization structure, not the typography of the numeral or the particular set-theoretic coding used to realize it.

Benacerraf, Structuralism, and Numerical Identity

Benacerraf (1965) observed that familiar set-theoretic constructions can represent the natural numbers by different sequences of sets. The von Neumann and Zermelo constructions, for example, assign different set-theoretic membership properties to corresponding numerical positions while supporting the same intended arithmetic structure. If arithmetic itself supplies no reason to privilege one such realization as giving the literal identity of the numbers, then an absolute identification of numbers with one chosen family of sets appears mathematically unmotivated.

The worked example makes the relevant distinction explicit. Two realizations may differ in their underlying objects while agreeing completely with respect to the arithmetic relations preserved by an isomorphism. From the standpoint of arithmetic, preserved structure is therefore more fundamental than the accidental features of a particular implementation.

Methodological Versus Metaphysical Structuralism

Two opposite conclusions should be avoided. The first is a naive reductionism that identifies numbers with one convenient set-theoretic realization merely because that realization supports arithmetic. The second is an overly strong structuralist inference according to which invariance under isomorphism, by itself, settles the ontology of numbers.

Benacerraf's argument undermines the first move more directly than it establishes the second. Logician, neo-logician, modal, set-theoretic, and other philosophical accounts can agree that arithmetic is invariant under isomorphism while disagreeing about what ultimately explains that invariance. The claim defended here is therefore comparative and methodological: mathematical practice strongly favors identifying numerical roles through preserved structure, but this does not by itself establish one uniquely correct metaphysics of numbers (Shapiro, 2000; Linnebo, 2017).

Why the Natural Numbers are a Favorable Case

The natural numbers provide an especially favorable setting for structural analysis because their positions are rigidly ordered by zero and successor. In the standard natural-number structure, any automorphism preserving 0 and successor must fix every natural number. Hence there is no nontrivial automorphism that permutes natural-number positions while preserving the full arithmetic structure.

A related but distinct point concerns categoricity. Under full second-order semantics, the second-order Dedekind-Peano axioms characterize the natural-number structure uniquely up to isomorphism. First-order Peano arithmetic is not categorical and has nonstandard models. This qualification matters: structural identification is especially

strong when the axioms and the relevant notion of structure determine the intended positions up to isomorphism (Shapiro, 2000; Linnebo, 2017).

The Limit of the Structural Inference

Even in this favorable case, structural characterization does not settle what a structure itself is or what kind of existence its positions possess. Realist, modal, eliminative, and set-theoretic forms of structuralism offer different answers. Harder identity questions also arise in non-rigid structures, where distinct positions may be structurally indiscernible under automorphisms. The natural numbers therefore provide strong evidence for a structural methodology, not a universal proof of one structuralist metaphysics.

Foundational Programs and the Limits of Reduction

The problem of numerical identity sits within a broader foundational debate about what arithmetic can be reduced to and what such a reduction would explain. Logicism, formalism, and intuitionism offered importantly different answers. Frege's logicist project sought to explain arithmetic through logic and definitions of number, thereby making the identity of numbers depend on a logical account rather than on arbitrary notation (Frege, 1959).

Formalist programs emphasized axiomatic systems, symbolic rules, and consistency, shifting attention from the intrinsic nature of mathematical objects toward the formal relations encoded by a theory. Brouwer's intuitionism rejected the idea that mathematics is exhausted by formal symbol manipulation and grounded mathematical objects in constructive mental activity (Brouwer, 1913). These programs disagree about ontology and epistemology, yet each illustrates the same methodological lesson: a successful formal representation is not automatically a complete philosophical explanation of the objects represented. Structural invariance can therefore constrain foundational theories without collapsing their differences.

Gödel and Formal Limitation

Gödel's incompleteness theorems impose precise limits on sufficiently expressive, consistent, effectively axiomatized formal systems. They should not be presented as showing that mathematics is uncertain or that formal methods fail. Their relevance here is narrower: mathematical knowledge about arithmetic cannot be identified, in the strongest simple sense, with derivability in one complete effective formal system (Gödel, 1931). This reinforces the need to distinguish formal representation, mathematical truth, and philosophical interpretation.

A Counterpoint: Cardinality and the Limits of Structural Description

The structural argument should also be tested against a possible objection. Some talk about numbers appears to concern cardinality directly: three apples and three sounds have the same cardinal number because there is a one-to-one correspondence between the collections. This may seem to provide content prior to any explicit arithmetic structure. Yet the objection does not undermine the present thesis. It shows instead that different mathematical contexts may foreground different invariant relations - equinumerosity in cardinal contexts, order and operations in arithmetic contexts.

Accordingly, the paper does not claim that one fixed list of arithmetic invariants exhausts every philosophical feature of number. Its narrower claim is that identity claims within a mathematical practice are governed by the relations that the practice treats as structure-preserving. Structural methodology is therefore context-sensitive without being arbitrary: the relevant morphisms and invariants are fixed by the mathematical structure under investigation (Carter, 2008; Schiemer, 2014).

Computer-Assisted Proof, Artificial Intelligence, and Mathematical Understanding

The distinction between preserved form and philosophical interpretation reappears at the epistemic level. A proof assistant can check that inferential steps satisfy a formal specification, and automated systems can search large spaces of candidate arguments. These achievements matter mathematically, but formal correctness alone does not

answer why a representation is fruitful, why a lemma is conceptually central, or why a proof reorganizes understanding.

Artificial intelligence extends the issue from verification toward search and discovery. AlphaGeometry, for example, demonstrated that an automated system can solve difficult olympiad-geometry problems within a restricted domain (Trinh et al., 2024). This supports a claim about automated mathematical reasoning, not the stronger claim that current AI systems possess mathematical understanding in the same sense as human mathematicians.

Truth, Verification, Explanation, and Understanding

Mathematical truth, reliable verification, explanation, and understanding are related but non-identical epistemic achievements. A machine may preserve formal relations perfectly while remaining silent about why a representation is illuminating or why a theorem matters. The same restraint used in moving from arithmetic invariance to ontology should therefore be used in moving from computational success to claims about mathematical understanding.

Table 2. Mathematical results and the philosophical questions they do -and do not- settle

Mathematical Fact	Philosophical Significance	Ontological Verdict
Different numerals can denote the same integer	Identity is not typography	Ontology remains open
Prime factorization is invariant	Arithmetic identity has stable relational content	Does not determine what numbers are
Isomorphisms preserve arithmetic properties	Supports methodological structuralism	Does not entail one metaphysics
Multiple realizations support the same arithmetic	Challenges naive set-identification	Competing accounts remain possible
Rigidity and second-order categoricity strengthen the case for $\mathbb{N}$	Natural-number positions are tightly structurally determined	A theory of structure is still required
Formal verification can be mechanized	Correctness can be separated from presentation	Understanding and explanation remain further questions

Scope and Limitations

This paper defends a conceptual thesis, not a comprehensive history and not a new theorem in number theory. Its examples are elementary because they make representation and invariance mathematically transparent. The argument is strongest for the natural numbers, whose intended structure is rigid, and should not be transferred without qualification to non-rigid structures. The historical discussion is selective, and the treatment of AI is epistemological rather than a general account of artificial intelligence. Finally, the argument supports methodological structuralism more directly than any particular metaphysical structuralism.

Conclusion

The paper began with a question of identity: what remains the same when numerical representations or realizations change? Decimal, binary, and Roman numerals can denote the same integer because notation is not numerical identity. At the arithmetic level, divisibility, primality, and factorization exhibit stable relational content. The worked isomorphism makes this precise: different underlying objects can support corresponding arithmetic roles. These facts support a structural description of mathematical practice. Number theory studies properties preserved under legitimate transformations, while foundational mathematics compares realizations through structure-preserving maps. Benacerraf's identification problem explains why implementation-specific set membership should not be confused with arithmetic identity.

The philosophical conclusion must nevertheless remain limited. Methodological structuralism does not entail a unique metaphysical structuralism. Mathematics determines which relations are preserved and which theorems hold, but it does not decide whether numbers are structural positions, logical constructions, objects instantiated in systems, or entities understood through another ontology. Arithmetic invariance constrains metaphysics without uniquely determining it.

Computer-assisted proof and AI reinforce the same distinction at the epistemic level. Machines can increasingly handle formal relations and verification, while explanation and understanding remain distinct questions. The common theme is disciplined abstraction: mathematics gains power by preserving structure across changes of realization, while philosophy asks what that preservation ultimately means.

Recommendations

Future work should test the distinction between methodological and metaphysical structuralism on mathematically richer cases. Non-unique factorization in algebraic number theory, structures with nontrivial automorphisms, and formally verified proofs are especially useful because they place pressure on simple accounts of identity, invariance, and understanding.

Scientific Ethics Declaration

* The authors declare that the scientific, ethical, and legal responsibility of this article belongs to the authors. This conceptual and historical study did not involve human participants, animals, personal data, or an intervention requiring ethics committee approval; therefore, ethics committee permission was not required.

Conflict of Interest

* The authors declare that they have no conflicts of interest.

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